

Aspects of Puff Field Theory

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ABSTRACT: We describe some features of the recently constructed “Puff Field Theory,” and present arguments in favor of it being a field theory decoupled from gravity. We construct its supergravity dual and calculate the entropy of this theory in the limit of large 't Hooft coupling. We also determine the leading irrelevant operator that governs its deviation from $\mathcal{N} = 4$ super Yang-Mills theory.

KEYWORDS: AdS-CFT Correspondence, Non-Commutative Geometry, Gauge-gravity correspondence.

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1. Introduction

The Melvin Universe is an exact solution of Einstein gravity coupled with gauge fields [1]. It describes a consistent gravitational backreaction when one attempts to support a uniform magnetic field in the background. (Of course, the magnetic field is no longer uniform when the gravitational backreaction is taken into account.) Melvin Universes are especially natural in the context of Kaluza-Klein theory. Simply consider twisting an angular coordinate so that the space-time, where a plane is expressed in cylindrical coordinates, has the form

$$ds^2 = -dt^2 + dx^2 + dr^2 + r^2(d\phi + \eta dz)^2 + dz^2 . \tag{1.1}$$

Here $z \sim z + 2\pi R$ is periodic, and η therefore cannot be trivially eliminated by a change of coordinates (since a redefinition $\phi \rightarrow \phi + \eta z$ would modify the periodicity conditions on z and ϕ). For $\eta = 0$, the space reduces to flat space in 4+1 dimensions. Kaluza-Klein reduction along the z coordinate gives rise to a space-time with background magnetic field and some background scalar field configuration.

This type of space-time has a natural embedding in string theory. Simply embed (1.1) in 10 or 11 dimensional supergravity. One concrete realization is to embed (1.1) in type

IIA supergravity. In this case, one can find an interesting type IIB supergravity solution by T-dualizing along the z direction. The type IIB supergravity solution is of the form

$$\begin{aligned}
 ds^2 &= -dt^2 + d\vec{x}^2 + dr^2 + \frac{r^2 d\phi^2}{1 + \eta^2 r^2} + \frac{1}{1 + \eta^2 r^2} dz^2 \\
 B &= \frac{\eta r^2}{1 + \eta^2 r^2} d\phi \wedge d\tilde{z} \\
 e^\Phi &= \sqrt{\frac{1}{1 + \eta^2 r^2}} \\
 \tilde{z} &\sim \tilde{z} + 2\pi\tilde{R}, \quad \tilde{R} = \frac{\alpha'}{R}.
 \end{aligned}
 \tag{1.2}$$

This background is not supersymmetric. However, it is straightforward to twist in more than one plane in such a way that some fraction of supersymmetry is preserved.

These background space-times, from the point of view of string theory, are special in that their world sheet sigma model is exactly solvable [2–7]. This follows, in essence, from the fact that they are dual to flat space with some periodic identifications.

Melvin universes are also useful for constructing non-local quantum field theories as a decoupling limit. Simply consider adding a D3-brane in the background (1.2) and take the appropriate scaling limit for the parameters η , r , and R as one sends α' to zero. In enumerating distinct embeddings of D3 branes in this background, it is useful to note that there are essentially two special spatial coordinates, \tilde{z} and ϕ . Taking both \tilde{z} and ϕ to be along the brane gives rise to non-commutative geometry with a position dependent non-commutativity parameter [8, 9] for which the deformation quantization formula of Kontsevich [10] becomes relevant. Taking \tilde{z} to be along the brane but ϕ to be transverse gives rise to a dipole deformation, which was introduced in [11, 12]. The construction by Lunin and Maldacena of β -deformed $\mathcal{N} = 4$ superconformal theory [13] can also be viewed as an example of this construction where both \tilde{z} and ϕ are transverse to the brane. Other constructions of a similar kind were studied in [14] and [15]. There are generalizations of these constructions that can be obtained by modifying the embeddings of the ϕ and the \tilde{z} coordinates in the full geometry, which were classified and tabulated in [8]. Most of these constructions give rise to a non-local field theory as a decoupling limit of string theories with solvable world sheet sigma model.

Recently, one of us proposed an example of a non-local field theory not included in [8], which arises from a novel embedding of a D-brane in a Melvin universe [16]. The construction proceeds as follows.

1. Start with N D0-branes in flat 10 dimensional background of type IIA string theory. The M-theory lift of this configuration corresponds to a Kaluza-Klein wave traveling along the M-theory circle. Let us parameterize the coordinates of the background geometry in 11 dimensions using coordinates

$$ds^2 = -dt^2 + \sum_{i=1}^3 dx_i^2 + \sum_{i=1}^3 (dr_i^2 + r_i^2 d\phi_i^2) + dz^2, \quad z \sim z + 2\pi R. \tag{1.3}$$

2. Twist the angular coordinate ϕ_i by a deformation parameter β_i so that the metric becomes

$$ds^2 = -dt^2 + \sum_{i=1}^3 dx_i^2 + \sum_{i=1}^3 [dr_i^2 + r_i^2(d\phi_i + \beta_i dz)^2] + dz^2. \quad (1.4)$$

3. Reduce back to type IIA along the z coordinate, giving rise to a D0-brane in a Melvin universe supported by a flux of magnetic RR 2-form field strength.
4. T-dualize along the $x_1, x_2,$ and x_3 directions. This will give rise to a configuration of D3-branes embedded in a Melvin universe with RR 5-form field strength in the background in type IIB theory.

The construction described in [16] started with a KK wave in type II theory but leads to the same geometry.

By taking a suitable scaling limit involving $\alpha', \beta_i,$ and the compactification radii as we will describe in more detail below, one arrives at a presumably decoupled system of non-local quantum field theory, similar in many regards to non-commutative Yang-Mills theory, dipole theory, and NCOS. This theory was named the ‘‘Puff Field Theory’’ (PFT) in [16], because the light degrees of freedom ‘‘puff up’’ in all three dimensions. The distinguishing feature of PFT is the fact that it leaves unbroken a spatial subgroup of the Lorentz group $SO(3) \subset SO(1, 3)$, unlike the more familiar non-commutative/dipole field theories. Such a theory could possibly be of phenomenological interest when applied to maximally isometric cosmological scenarios of the Freedman-Robertson-Walker type.

The goal of this article is to describe the features of PFT and to provide more evidence in support of the conjecture that PFT is decoupled from gravity. This is facilitated by the explicit construction of the supergravity dual. This paper is organized as follows. In section 2, we describe the construction of the supergravity dual itself. In section 3, we construct the finite temperature case and describe the thermodynamics of PFT. In section 4, we describe the RG flow of PFT in greater detail. In section 5, we identify the leading irrelevant operator responsible for deforming $\mathcal{N} = 4$ SYM to PFT. We conclude in section 6.

2. Supergravity dual of PFT

In this section, we describe the construction of the supergravity solution that is dual to PFT. In what follows, we present the steps leading to the solution (2.20) in some detail. After that, we will analyze the regime of validity of the classical supergravity solution.

To obtain the supergravity dual of PFT, we start from type IIA supergravity compactified along the x_i directions and consider N D0-branes smeared along the compact directions. In anticipation of the T -duality, we will denote the period of compactification

of x_i by α'/R_i . This geometry can be written explicitly in the form

$$\begin{aligned}
 ds^2 &= -h^{-1/2}dt^2 + h^{1/2} \left(\sum_{i=1}^3 dx_i^2 + \sum_{i=4}^9 dy_i^2 \right) \\
 A &= h^{-1}dt \\
 e^{2\Phi} &= h^{3/2} .
 \end{aligned}
 \tag{2.1}$$

where

$$h = 1 + \frac{60\pi^3 g_{\text{IIA}} N \alpha'^{7/2}}{(\|x\|^2 + \|y\|^2)^{7/2}}
 \tag{2.2}$$

for the “non-smearred” solution (for which directions 1, 2, 3 are noncompact and $-\infty < x_1, x_2, x_3 < \infty$), and

$$h = 1 + \frac{4\pi g_{\text{IIA}} N R_1 R_2 R_3 \alpha'^{1/2}}{\|y\|^4}
 \tag{2.3}$$

for the “smearred” solution (which is obtained from the “non-smearred” one by replacing the second term in h with its integral over the full range of x_1, x_2, x_3 and dividing by the total volume $(2\pi\alpha')^3/R_1 R_2 R_3$ of the T^3). Here

$$\|x\|^2 \equiv \sum_{i=1}^3 x_i^2, \quad \|y\|^2 \equiv \sum_{i=4}^9 y_i^2 .
 \tag{2.4}$$

Now we perform the M-theory lift, twist, and reduction back to type IIA. The M-theory lift of (2.1) is purely geometric

$$ds^2 = -h^{-1}dt^2 + h(dz - h^{-1}dt)^2 + \left(\sum_{i=1}^3 dx_i^2 + \sum_{i=4}^9 dy_i^2 \right) .
 \tag{2.5}$$

Here z is a periodic coordinate (the “M-circle”) with period $z \sim z + 2\pi g_{\text{IIA}} \ell_s$ (where $\ell_s \equiv \alpha'^{1/2}$ is the string scale). One can in principle consider letting all β_i ’s take independent values in performing the twist. We will, however, concentrate on the case where $\beta_1 = \beta_2 = \eta$ and $\beta_3 = 0$, which leaves half of the supersymmetries unbroken, and will at the end give us a theory with $\mathcal{N} = 2$ in 4D. Then, the metric after the twist has the form

$$ds^2 = -h^{-1}dt^2 + h(dz - h^{-1}dt)^2 + \sum_{i=1}^3 dx_i^2 + \sum_{i=1}^2 [dr_i^2 + r_i^2(d\phi_i + \eta dz)^2] + \sum_{i=8}^9 dy_i^2 .
 \tag{2.6}$$

Before we proceed, we make another convenient change of variables. We replace the four coordinates r_1, r_2, ϕ_1, ϕ_2 with the radial variable

$$u \equiv \sqrt{r_1^2 + r_2^2}, \quad (0 \leq u < \infty),
 \tag{2.7}$$

and three new angular coordinates ϕ, θ, φ defined as follows,

$$\varphi \equiv \phi_1 - \phi_2, \quad \sin \theta \equiv \frac{2r_1 r_2}{r_1^2 + r_2^2}, \quad \phi \equiv \phi_1 .
 \tag{2.8}$$

For fixed $t, x_1, x_2, x_3, y_8, y_9$, and u , the variables ϕ, θ, φ describe an S^3 in the form of a Hopf fibration: (θ, φ) are spherical coordinates on the S^2 base, and ϕ is a periodic coordinate on the S^1 fiber with period 2π . Presenting S^3 as a Hopf fibration is convenient because the twist is in the direction of the fiber ϕ . In order to save space, we denote the (Fubini-Study) metric on the base of the Hopf fibration by

$$ds_{B(2)}^2 \equiv \frac{1}{4}(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2.9)$$

and we denote the connection of the Hopf fibration by

$$\mathcal{A} \equiv -\frac{1}{2}(1 - \cos \theta)d\varphi. \quad (2.10)$$

The metric can now be expressed in the form

$$ds^2 = -h^{-1}dt^2 + h(dz - h^{-1}dt)^2 + \sum_{i=1}^3 dx_i^2 + du^2 + u^2 \left[ds_{B(2)}^2 + (d\phi + \eta dz + \mathcal{A})^2 \right] + \sum_{i=8}^9 dy_i^2. \quad (2.11)$$

Reducing along dz to IIA then gives

$$ds^2 = k^{1/2} \left(-h^{-1}dt^2 + \sum_i dx_i^2 + du^2 + u^2 ds_{B(2)}^2 + \sum_{i=8,9} dy_i^2 \right) + k^{-1/2} h u^2 (d\phi + \mathcal{A} + \eta h^{-1}dt)^2, \\ A = k^{-1} (-dt + u^2 \eta (d\phi + \mathcal{A})), \\ e^\Phi = g_{\text{IIA}} k^{3/4}, \quad (2.12)$$

where

$$k \equiv h + \eta^2 u^2 = 1 + \frac{4\pi g_{\text{IIA}} N R_1 R_2 R_3 \alpha'^{1/2}}{(u^2 + \|y\|^2)^2} + \eta^2 u^2. \quad (2.13)$$

T-dualizing along x_1, x_2 , and x_3 then gives rise to the supergravity solution

$$ds^2 = k^{1/2} \left(-h^{-1}dt^2 + du^2 + u^2 ds_{B(2)}^2 + \sum_{i=8,9} dy_i^2 \right) + k^{-1/2} \left(\sum_i dx_i^2 + h u^2 (d\phi + \mathcal{A} + \eta h^{-1}dt)^2 \right), \\ A = k^{-1} (-dt + u^2 \eta (d\phi + \mathcal{A})) \wedge dx_1 \wedge dx_2 \wedge dx_3, \\ e^\Phi = g_{\text{IIB}} = R_1 R_2 R_3 \alpha'^{-3/2} g_{\text{IIA}}. \quad (2.14)$$

Here A is not quite the full Ramond-Ramond gauge 4-form field, but the full Ramond-Ramond 5-form field strength is the self-dual part of dA . In the new variables, h and k take the forms

$$h = 1 + \frac{4\pi g_{\text{IIB}} N \alpha'^2}{(u^2 + \|y\|^2)^2}, \quad k = 1 + \frac{4\pi g_{\text{IIB}} N \alpha'^2}{(u^2 + \|y\|^2)^2} + \eta^2 u^2. \quad (2.15)$$

Note that if we set $N = 0$ we get $h = 1$ and the supergravity solution reduces to a Melvin universe with background RR 5-form flux. For $N \neq 0$, The warping due to $h(u, y)$ describes the gravitational back-reaction of the D3-branes.

Now, we can take the decoupling limit following the procedure of [17]. As usual, in order to decouple the gauge theory from the rest of the string theory modes, we take the zero slope limit $\alpha' \rightarrow 0$, and we need to specify how to scale the coordinates y_8, y_9, u and the twist parameter η in this limit. It turns out that the appropriate scaling is to keep finite the following rescaled coordinates

$$U \equiv \alpha'^{-1}u, \quad Y_i \equiv \alpha'^{-1}y_i \quad (i = 8, 9), \quad (2.16)$$

while scaling the twist parameter η so that

$$\Delta^3 \equiv \eta\alpha'^2 = \text{fixed}, \quad (2.17)$$

and keeping $g_{\text{IIB}} = 2\pi g_{\text{YM}}^2$, R_1 , R_2 , and R_3 fixed. This scaling is chosen so that in the dual supergravity solution the effects of the deformation will be finite in the scaling limit. For example, the second and third terms in k in (2.15) are comparable in this scaling limit. Our scaling limit also turns out to be the one suggested in [16] using different arguments.

Note in particular that this is the scaling that keeps the angle of twist per radius of the M-circle,

$$\chi \equiv g_{\text{IIA}}\ell_s\eta = \frac{\eta g_{\text{IIB}}\alpha'^2}{R_1 R_2 R_3} = \frac{2\pi g_{\text{YM}}^2 \Delta^3}{R_1 R_2 R_3}, \quad (2.18)$$

finite. We will see below that an integer shift in χ leads to an equivalent theory up to a certain duality, similarly to the structure of Morita equivalence encountered in the context of ordinary non-commutative field theories. We are now almost ready to take the $\alpha' \rightarrow 0$ limit. We define the scaled harmonic functions (with the notation $\|Y\|^2 \equiv Y_8^2 + Y_9^2$)

$$H \equiv \lim_{\alpha' \rightarrow 0} \alpha'^2 h = \frac{4\pi g_{\text{IIB}} N}{(U^2 + \|Y\|^2)^2}, \quad K \equiv \lim_{\alpha' \rightarrow 0} \alpha'^2 k = \frac{4\pi g_{\text{IIB}} N}{(U^2 + \|Y\|^2)^2} + \Delta^6 U^2, \quad (2.19)$$

which captures the asymptotic behavior of the harmonic functions h and k in the decoupling limit, and depend only on the PFT parameters $g_{\text{IIB}} \equiv 2\pi g_{\text{YM}}^2$ and Δ . In terms of H and K we can write

$$\begin{aligned} \frac{ds^2}{\alpha'} &= K^{1/2} \left(-H^{-1} dt^2 + dU^2 + U^2 ds_{B(2)}^2 + \sum_{i=8,9} dY_i^2 \right) \\ &\quad + K^{-1/2} \left(\sum_i dx_i^2 + HU^2 (d\phi + \mathcal{A} + \Delta^3 H^{-1} dt)^2 \right), \\ \frac{A}{\alpha'^2} &= K^{-1} (-dt + U^2 \Delta^3 (d\phi + \mathcal{A})) \wedge dx_1 \wedge dx_2 \wedge dx_3, \\ e^\Phi &= g_{\text{IIB}} = 2\pi g_{\text{YM}}^2. \end{aligned} \quad (2.20)$$

This is an exact solution of the classical equations of motion of type IIB supergravity, and we will interpret it as the supergravity dual of PFT. Much of the conclusions we draw regarding the nature of PFT will be based on this supergravity solution, which is one of the main results we are reporting in this paper.

PFT depends on a dimensionful parameter Δ , which according to its definition in (2.17), has dimensions of length. For generic Δ , the solution (2.20) is invariant under Poincaré transformations in the t, x_1, x_2, x_3 directions, under $SU(2)$ rotations of the base $B(2)$ of the Hopf fibration (acting on the spin structure and the fiber direction ϕ as well), under $U(1)$ translations generated by the vector field $\partial/\partial\phi$, and under 8 supersymmetries. For $\Delta = 0$ the solution (2.20) reduces to $AdS_5 \times S^5$ – the coordinates t, x_1, x_2, x_3 and

$$V \equiv \sqrt{U^2 + \|Y\|^2} \tag{2.21}$$

parameterize the AdS_5 part. (V can be traced back to the radial direction transverse to the D3-brane.) For later use, it is also convenient to define the angle $0 \leq \psi \leq \pi/2$ by

$$U = V \cos \psi, \quad \|Y\| = V \sin \psi . \tag{2.22}$$

Up to factors of certain powers of $\lambda = 2\pi g_{YM}^2 N = g_{IIB} N$ that will be discussed later, Δ^{-1} sets the interesting energy scale for PFT. This is the scale above which PFT becomes appreciably different from $\mathcal{N} = 4$ SYM. This can be seen directly from (2.20): for $U \ll \Delta^{-1}$ (and fixed λ), the solution asymptotes to $AdS_5 \times S_5$, indicating that the infra-red fixed point of this theory is $\mathcal{N} = 4$ SYM. On the other hand, for $U \gg \Delta^{-1}$, the solution deviates strongly from the $AdS_5 \times S_5$ background. The supergravity duals of other nonlocal field theories such as non-commutative Yang-Mills theory [18, 19] and dipole theory [12] also exhibit similar features.

Regime of validity. Let us comment on the region of validity of the dual supergravity description. The 't Hooft coupling constant $\lambda = 2\pi g_{YM}^2 N = g_{IIB} N$ must be large in order for the curvature to be weak. We assume that the Yang-Mills coupling constant g_{YM} itself is kept finite. (Of course, g_{YM} has to be small if one wishes to extend the discussion beyond the classical supergravity description, for example, to include the excited string spectrum.)

If $\lambda \gg 1$, the curvature is small everywhere provided $\psi \neq \pi/2$. More specifically, the invariant square of the curvature tensor, as calculated from the string-frame metric (2.20), is

$$\begin{aligned} \mathcal{R}_{\mu\nu\sigma\tau} \mathcal{R}^{\mu\nu\sigma\tau} = & \alpha'^{-2} (4\pi\lambda + \Delta^6 U^2 V^4)^{-5} \left[80(4\pi\lambda)^4 - 80(4\pi\lambda)^3 \Delta^6 V^4 (3U^2 + 5V^2) \right. \\ & + 24(4\pi\lambda)^2 \Delta^{12} V^8 (136U^4 + 29U^2 V^2 + 5V^4) \\ & \left. + 32\pi\lambda \Delta^{18} V^{12} U^2 (15V^4 + 7U^2 V^2 - 72U^4) + 65\Delta^{24} U^4 V^{20} \right] . \end{aligned} \tag{2.23}$$

So, if $\psi \neq \pi/2$ we see that for $U\Delta \ll \lambda^{1/6}$ the curvature scale is of order $\alpha'^{-1/2} \lambda^{-1/4}$, while for $U\Delta \gg \lambda^{1/6}$ the curvature scale is of order $\alpha'^{-1/2} (U\Delta)^{-3/2}$. Both of these quantities are small for $\lambda \gg 1$. If $\psi = \pi/2$, on the other hand, the curvature is small only for $\Delta\|Y\| \ll \lambda^{3/4}$. Therefore, observables that sensitively probe the $\psi = \pi/2$ region might receive corrections due to stringy effects.

Another requirement for the classical supergravity analysis to be applicable is that the proper size of the various compact directions be large compared to the string scale. For the ϕ direction we get

$$K^{-1/2} H U^2 \gg 1. \tag{2.24}$$

Assuming again that $\psi \neq \pi/2$, we find that the ϕ direction is large if $U\Delta \ll \lambda^{1/3}$. On the other hand, if $U\Delta \gg \lambda^{1/3}$ the ϕ -circle is smaller than string scale and its radius is of order $\alpha^{1/2}\lambda^{1/2}(U\Delta)^{-3/2}$. In this regime the supergravity dual (2.20) cannot be trusted. For $\psi \neq \pi/2$, the radius of the ϕ -circle shrinks to zero, but the solution is not singular. To see this, note that for fixed nonzero $\|Y\|$ we have $V \neq 0$, and as $\psi \neq \pi/2$ (and therefore $U \rightarrow 0$) the metric on the base $ds_{B(2)}^2$ and the fiber ϕ combine to a metric on S^3 , and together with the U direction we get the metric on a ball.

Next, we need to discuss directions x_1, x_2, x_3 . The proper size of each of these compact directions needs to be large in comparison to string scale. One way to achieve this is to simply take the decompactification limit $R_i \rightarrow \infty$ ($i = 1, 2, 3$) and formulate PFT on $\mathbb{R}^{3,1}$, so to speak. Alternatively, we can keep the compactification radii R_1, R_2, R_3 finite. Taking this approach, as we shall see in section 4, yields a richer structure of energy scales in the theory, but then in order for the classical supergravity solution (2.20) to be valid, we need the additional requirements

$$R_i \gg K^{1/4}, \quad i = 1, 2, 3. \tag{2.25}$$

Assuming that $\psi \neq \pi/2$, we find that K is of the order of $4\pi\lambda U^{-4} + \Delta^6 U^2$. This expression is never smaller than $3(\pi\lambda)^{1/3}\Delta^4$, and therefore (2.25) will not be satisfied unless

$$R_i \gg \lambda^{1/12}\Delta, \quad (i = 1, 2, 3). \tag{2.26}$$

Assuming (2.26) now, the condition (2.25) sets the following range requirement for U :

$$\frac{\lambda^{1/4}}{R_i} \ll U \ll \frac{R_i^2}{\Delta^3}. \tag{2.27}$$

The validity conditions that we found so far can be recast in terms of the energy scale. For $AdS_5 \times S^5$, the holographic energy/distance relation [20] takes the form

$$E = \frac{U}{\sqrt{\lambda}}. \tag{2.28}$$

In order of magnitude, this form is also applicable to our metric, at least if we assume that $\cos\psi$ is of order $O(1)$, so that U and V are comparable. We will demonstrate this later in (3.6) of section 3, when we discuss PFT at nonzero temperature.

The conditions (2.27) can now be written as a range of energy scales

$$\frac{\lambda^{-1/4}}{R_i} \ll E \ll \frac{\lambda^{-1/2}R_i^2}{\Delta^3}, \tag{2.29}$$

and the condition about the ϕ -circle that was found above becomes

$$E \ll \frac{\lambda^{-1/6}}{\Delta}. \tag{2.30}$$

Combining (2.30) and (2.29) we get

$$\frac{\lambda^{-1/4}}{R_i} \ll E \ll \min\left\{\frac{\lambda^{-1/6}}{\Delta}, \frac{\lambda^{-1/2}R_i^2}{\Delta^3}\right\}. \tag{2.31}$$

Similar sets of bounds on the region of validity for the case of non-commutative Yang-Mills theory were pointed out in [21]. Note also that, assuming (2.26) and $\lambda \gg 1$, we have the inequality

$$\frac{\lambda^{-1/4}}{R_i} \ll \frac{\lambda^{-1/3}}{\Delta} \ll \min \left\{ \frac{\lambda^{-1/6}}{\Delta}, \frac{\lambda^{-1/2} R_i^2}{\Delta^3} \right\}. \quad (2.32)$$

The energy scale $\lambda^{-1/3}/\Delta$ is important because it corresponds to $U\Delta = \lambda^{1/6}$, which is the scale at which the metric starts to deviate markedly from $AdS_5 \times S^5$. For example, below that scale $H \approx K$. This is therefore the scale at which PFT effects enter into play, and we see from (2.32) that it is inside the range of validity (2.31).

In (2.31), the lower bound $\lambda^{-1/4}/R_i \ll E$ is independent of the non-locality and applies just as well to the case of ordinary $AdS_5 \times S^5$ compactified on a circle. The bound simply indicates the presence of finite size effects cutting off the spectrum in the IR. The order of magnitude of the size of a typical excitation with energy E can be estimated as the Compton length $L = 1/E$, but this estimate fails when the size of the excitation gets bigger than the size of the box. This explains why the lower bound on E is proportional to $1/R_i$. The factor of $\lambda^{-1/4}$ in the bound is the effect of strong coupling. For energies below the bound, $E \ll \lambda^{-1/4}/R_i$, one should look for a description in terms of the near horizon geometry of a lower dimensional brane. Readers are referred to [22, 23] for explanations concerning the correct cross-over behavior and the correspondence principle at work around this scale.

The upper bound on E in (2.29) implies that the size of a typical excitation starts to grow with energy according to $L \sim \sqrt{\lambda^{1/2} \Delta^3 E}$, so that the upper bound is reached when $L \sim R_i$. This is a characteristic feature of non-local field theories. The size of an object grows both in the extreme IR and in the extreme UV. When the size of the object becomes larger than the size of the box, one must adopt an alternative description. We will comment further on this issue in section 4.

The high energy regime $E \gg \lambda^{-1/6}/\Delta$. So far, we discussed the upper bound on energy coming from (2.29), but we also have another upper bound from (2.30). The latter suggests another interesting length-scale in the problem, namely $\lambda^{1/6}\Delta$, at least for strong 't Hooft coupling $\lambda \gg 1$. As we reach the corresponding range $U \sim \lambda^{1/3}/\Delta$ in the supergravity solution (2.20) the ten-dimensional description loses its classical interpretation. If the compactification radii R_1, R_2 and R_3 are all much bigger than the length scale $\lambda^{1/6}\Delta$, then

$$\frac{\lambda^{1/3}}{\Delta} \ll \frac{R_i^2}{\Delta^3}, \quad (2.33)$$

and the energy scale E (or equivalently, U) at which the ϕ -circle becomes comparable to the string length-scale is lower than the energy scale at which the radii of the x_1, x_2, x_3 fall below the string scale. In particular, this is the case in the decompactification limit $R_i \rightarrow \infty$. There is then a range of U for which, even though the ϕ -circle is small, we can still dimensionally reduce along it to get a valid nine-dimensional classical supergravity description, as long as we keep away from the $\psi = \pi/2$ locus, near which the size of the fiber varies rapidly.

In the extreme regime $U \gg \lambda^{1/3}/\Delta$ the ϕ -circle is smaller than string scale, and it therefore makes sense to apply a T-duality transformation, at least away from the $\psi = \pi/2$ locus. Using the formulas of [24] we arrive at a background with the following NSNS fields:

$$\begin{aligned} \frac{ds^2}{\alpha'} &= K^{1/2} \left(-H^{-1} dt^2 + dU^2 + U^2 ds_{B(2)}^2 + \sum_{i=8,9} dY_i^2 \right) \\ &\quad + K^{-1/2} \sum_i dx_i^2 + K^{1/2} H^{-1} U^{-2} d\tilde{\phi}^2, \\ \frac{1}{\alpha'} B^{(\text{NSNS})} &= d\tilde{\phi} \wedge (\mathcal{A} + \Delta^3 H^{-1} dt), \\ e^\Phi &= 2\pi g_{\text{YM}}^2 K^{1/4} H^{-1/2} U^{-1}. \end{aligned} \tag{2.34}$$

Here $\tilde{\phi}$ is a periodic variable with period 2π that parameterizes the T-dual circle, and there are also nonzero RR fields that have not been written down here, for simplicity. (See also [25] for a related discussion where T-duality has been applied to $AdS_5 \times S^5$ by viewing the S^5 as a Hopf fibration over a base CP^2 .) There are, however, at least three extra complications:

1. The physics at the vicinity of the locus $\psi = \pi/2$ is not captured properly by (2.34). As we will now explain, the strongly curved metric in that region should be replaced by an NS5-brane. To see this, first note that for fixed $t, x_1, x_2, x_3, \tilde{\phi}$ and fixed $V \neq 0$, the remaining parts of the metric

$$K^{1/2} \left(dU^2 + U^2 ds_{B(2)}^2 + \sum_{i=8,9} dY_i^2 \right) \tag{2.35}$$

describe a space that is topologically equivalent to an S^4 . In fact, defining a new periodic coordinate $0 \leq \xi < 2\pi$ by

$$Y_8 = \|Y\| \cos \xi, \quad Y_9 = \|Y\| \sin \xi, \tag{2.36}$$

our piece of the metric reduces to

$$K^{1/2} V^2 \left(d\psi^2 + \frac{1}{4} \cos^2 \psi (d\theta^2 + \sin^2 \theta d\varphi^2) + \sin^2 \psi d\xi^2 \right), \tag{2.37}$$

where we used (2.9). And were it not for the explicit dependence of K on ψ and for the factor of $1/4$ in front of the second term, the metric (2.37) would describe an S^4 exactly. We are now ready to analyze the region near $\psi = \pi/2$. The locus $\psi = \pi/2$ is an S^1 (that can be parameterized by ξ), and the (θ, φ) variables describe an S^2 that shrinks to zero as $\psi \rightarrow \pi/2$. The radius of the $\tilde{\phi}$ -circle, on the other hand, increases indefinitely. As we approach $\psi = \pi/2$, however, the flux of the NSNS 3-form field-strength through the $S^2 \times S^1$ (generated by θ, φ and $\tilde{\phi}$) remains finite:

$$\frac{1}{\alpha'} \int_{S^2 \times S^1} dB^{(\text{NSNS})} = d\tilde{\phi} \wedge d\mathcal{A} = 4\pi^2.$$

This indicates the presence of one unit of NS5-brane charge at $\psi = \pi/2$.

2. When applying T-duality in superstring theory to a background that is a circle fibration, one has to be careful about the boundary conditions for fermions along the fiber direction. Specifically, before the T-duality, consider the holonomy for a closed path that wraps the fiber over a fixed point in the base. Assuming that the fiber varies slowly over the base, the geometrical holonomy is close to the identity in $SO(10)$, but when lifted to spinors the holonomy could be close to either $(+1)$ or (-1) in $Spin(10)$. We can determine which case corresponds to our metric by noting that the fiber of the Hopf fibration S^1 (the ϕ -circle) over S^2 (the θ, φ base) is contractible, and its spin holonomy can therefore be calculated unambiguously, and it is easy to see that it is (-1) . Note, however, that the full holonomy of fermions along the fiber is $(+1)$, because our solution preserves supersymmetry. The minus sign from the geometrical spin holonomy is canceled by the nongeometrical contribution to the holonomy due to the RR 5-form field strength.

3. Finally, we note that the dilaton in (2.34) gets large for

$$U \sim \frac{\lambda^{1/3}}{g_{\text{IIB}}^{2/3} \Delta} \sim \frac{N^{2/3}}{\lambda^{1/3} \Delta}, \tag{2.38}$$

assuming $\psi \neq \pi/2$, as usual. This scale of U is larger than the bound $\lambda^{1/3}/\Delta$, because we are always taking N to be very large. Thus, (2.34) is likely to be valid in a range

$$\lambda^{1/3}/\Delta \ll U \ll \frac{N^{2/3}}{\lambda^{1/3} \Delta}. \tag{2.39}$$

The upper bound (2.38) is smaller than the upper bound of (2.27) if

$$\Delta < N^{-1/3} \lambda^{1/6} R_i. \tag{2.40}$$

For finite R_i , this is never the case, but in the decompactification limit $R_i \rightarrow \infty$ this holds. Then, for $U \gg N^{2/3} \lambda^{-1/3}/\Delta$ the dilaton becomes large, and a proper description requires 11-dimensional supergravity. Lifting the solution (2.34) to M-theory, we get the metric

$$\begin{aligned} \frac{ds^2}{\ell_p^2} = N^{2/3} \lambda^{-2/3} & \left[K^{1/3} H^{1/3} U^{2/3} \left(-H^{-1} dt^2 + dU^2 + U^2 ds_{B(2)}^2 + \sum_{i=8,9} dY_i^2 \right) \right. \\ & \left. + K^{-2/3} H^{1/3} U^{2/3} \sum_i dx_i^2 + K^{1/3} H^{-2/3} U^{-4/3} d\tilde{\phi}^2 \right] \\ & + N^{-4/3} \lambda^{4/3} K^{1/3} H^{-2/3} U^{-4/3} dy_{10}^2, \end{aligned} \tag{2.41}$$

where $0 \leq y_{10} < 2\pi$ is a new periodic coordinate. For fixed $\psi \neq \pi/2$ and $U \gg \lambda^{1/6}/\Delta$, we may approximate

$$H \approx 4\pi \lambda U^{-4} \cos^4 \psi, \quad K \approx \Delta^6 U^2, \tag{2.42}$$

and

$$\frac{ds^2}{\ell_p^2} \approx \left(\frac{N}{4\pi\lambda^2 \cos^4 \psi} \right)^{2/3} \left[-U^4 \Delta^2 dt^2 + 4\pi\lambda \cos^4 \psi U^{-2} \Delta^{-4} \sum_i dx_i^2 + U^2 \Delta^2 d\tilde{\phi}^2 + 4\pi\lambda \Delta^2 \cos^4 \psi \left(dU^2 + U^2 ds_{B(2)}^2 + \sum_{i=8,9} dY_i^2 \right) + N^{-2} \lambda^2 \Delta^2 U^2 dy_{10}^2 \right]. \quad (2.43)$$

There is also a nonzero 3-form that we will not write down. It is interesting to note that as $U \rightarrow \infty$, the metric (2.43) becomes more and more flat (for $\psi \neq \pi/2$), and this suggests that the ultrahigh energy regime of noncompact PFT (all $R_i = \infty$) is holographically dual to a weakly coupled M-theory background. On the other hand, if R_i is finite the size of the x_i direction in (2.43) becomes comparable to the Planck scale for

$$U \sim \frac{\lambda^{-1/6} N^{1/3} R_i}{\Delta^2}. \quad (2.44)$$

Beyond that scale, the lift to M-theory is insufficient, and more complicated duality transformations are in order. There is in fact an intricate phase structure depending sensitively on the rationality of χ (or how well it is approximated by a rational number with a given denominator), which we will study in detail in section 4.

3. Thermodynamics of Puff Field Theory

A simple observable one can compute from the supergravity dual is the entropy. The entropy as a function of temperature can be extracted from the finite temperature generalization of the dual supergravity solution by applying the Beckenstein-Hawking formula [26, 27]. The finite temperature solution is also easy to construct for PFT. One simply starts with the smeared non-extremal D0-brane solution instead of (2.1) which has the following form:

$$ds^2 = -f h^{-1/2} dt^2 + h^{1/2} \sum_{i=1}^3 dx_i^2 + h^{1/2} f^{-1} d\rho^2 + \rho^2 (d\psi^2 + \sin^2 \psi d\xi^2 + \cos^2 \psi [ds_{B(2)}^2 + (d\phi + \mathcal{A})^2])$$

$$A = h^{-1} dt$$

$$e^{2\Phi} = h^{3/2} \quad (3.1)$$

where

$$f \equiv 1 - \frac{\rho_0^4}{\rho^4} \quad (3.2)$$

is the ‘‘thermal factor,’’ and $ds_{B(2)}^2$ and \mathcal{A} were defined in (2.9) and (2.10), respectively. Applying the same set of transformations, we arrive at a solution

$$\frac{ds^2}{\alpha'} = K^{1/2} \left(-f H^{-1} dt^2 + f^{-1} dV^2 + V^2 (\cos^2 \psi ds_{B(2)}^2 + \sin^2 \psi d\xi^2 + d\psi^2) \right) + K^{-1/2} \left(\sum_i dx_i^2 + H V^2 \cos^2 \psi (d\phi + \mathcal{A} + \Delta^3 H^{-1} dt)^2 \right),$$

$$\frac{A}{\alpha'^2} = K^{-1} (-dt + V^2 \Delta^3 \cos^2 \psi (d\phi + \mathcal{A})) \wedge dx_1 \wedge dx_2 \wedge dx_3,$$

$$e^\Phi = g_{\text{IIB}} = 2\pi g_{\text{YM}}^2, \quad (3.3)$$

where

$$f = 1 - \frac{V_0^4}{V^4}, \tag{3.4}$$

and

$$H = \frac{4\pi g_{\text{IIB}} N}{V^4}, \quad K = H + \cos^2 \psi \Delta^6 V^2, \tag{3.5}$$

in accordance with (2.19). Here V_0 is a free parameter, and the background (3.3) reduces to (2.20) for $V_0 = 0$. Similar constructions of non-extremal solutions in asymptotically non-trivial geometries have also appeared in [28].

The $V = V_0$ hypersurface corresponds to the horizon in this geometry. In order to extract the thermodynamic behavior of entropy $S(T)$ as a function of temperature, it is useful to first determine the temperature T and the horizon area A as a function of the horizon radius V_0 . As usual, the Hawking temperature can be inferred from the condition that the Euclidean continuation of this solution be singularity-free. This gives

$$V_0 = 2\pi\sqrt{\pi\lambda}T, \tag{3.6}$$

which is also one of the standard derivations of the energy-distance relation (2.28).

In order to apply the Beckenstein-Hawking formula, we need the area of the horizon in the Einstein frame. In string frame, we need

$$A = \frac{1}{\alpha'^4} e^{-2\Phi} \sqrt{g_s}, \tag{3.7}$$

where g_s is the determinant of the induced metric on the horizon. This formula, applied to (3.3), gives

$$S = \frac{1}{g_{\text{IIB}}^2} \sqrt{g_{\text{IIB}} N} (2\pi)^3 \mathcal{V} V_0^3 = N^2 (2\pi)^3 \mathcal{V} T^3, \tag{3.8}$$

where $\mathcal{V} = R_1 R_2 R_3$. This is our main conclusion concerning the entropy. Note that the final expression is independent of the ‘‘puffness’’ Δ . It should be emphasized, however, that this result is reliable only in the range of temperatures

$$\frac{\lambda^{-1/4}}{R_i} \ll T \ll \frac{\lambda^{-1/2} R_i^2}{\Delta^3}, \tag{3.9}$$

which does depend on the puffness. This is similar to what was found in non-commutative gauge theory [29, 19, 21]. We will comment further on the implication of the range of validity in section 4.

The metric (3.3) also contains information about the chemical potential conjugate to R-charge. Like PFT itself, the metric preserves an $SU(2) \times U(1)$ subgroup of the R-symmetry group $SU(4)$. The $U(1)$ component is generated by $\partial/\partial\phi$, and the chemical potential conjugate to the corresponding R-charge can easily be read-off from the solution (3.3). It is the angular velocity of the event-horizon:

$$\mu_R \equiv \Omega_{\text{horizon}} = \Delta^3 H^{-1} \Big|_{\text{horizon}} = \frac{\eta\alpha'^2}{4\pi g_{\text{IIB}} N} V_0^4 = 4\pi^5 \lambda \Delta^3 T^4. \tag{3.10}$$

The chemical potential depends on the puffness Δ , and it would be interesting to find the holographic dual for zero chemical potential, for which the entropy might also depend on Δ . We hope to report on this in a separate paper.

4. Renormalization group flow and hierarchy of PFT

The supergravity solution (2.20) is reliable in the range (2.27). It is natural to contemplate what alternative description takes over as a reliable description outside this range. Precisely such an issue, in the context of non-commutative gauge theory, was investigated in [21]. We will see below that the PFT case is quite similar.

The infra-red boundary of the region of validity (2.27) has a simple interpretation: at sufficiently low energies the higher dimensional operators deforming the theory become irrelevant, and one simply undergoes dimensional reduction below the scale of the size of the compactification. Let us assume for simplicity that R_1 , R_2 and R_3 are of the same order of magnitude. From the string theory dual point of view, T-duality along the compact directions maps the D3-branes to D0-branes. One expects the Gregory-Laflamme instability to localize the smeared supergravity solution, simply giving rise to the near horizon geometry of the D0-branes as the effective description beyond the region of validity (2.27) on the IR side [22, 23].

The proper size of the compact direction also becomes sub-stringy at the other end of the region of validity (2.27), i.e., at the upper bound on U . One does not expect the same three T-dualities to transform this background to a description which is effective. The lesson from non-commutative gauge theories and NCOS prompts us to look for more complicated U-duality transformations that can make that region of the background weakly coupled. In the case of non-commutative gauge theories and NCOS, the appropriate transformations are elements of $SL(2, Z)$, namely T- and S-duality transformations, respectively [21, 30]. For PFT, we propose the following $SL(2, Z)$ transformation:

1. First, T-dualize along x_1 , x_2 , and x_3 and lift to M-theory. This brings us back to (2.11) where z is a periodic coordinate with radius

$$R = g_{\text{IIA}} \ell_s . \tag{4.1}$$

2. Now, perform a coordinate transformation

$$\begin{pmatrix} d\phi \\ \frac{dz}{R} \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d\phi \\ \frac{dz}{R} \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, Z), \tag{4.2}$$

which makes the metric take the form

$$\begin{aligned} ds^2 = & -h^{-1}dt^2 + h(dz + cRd\phi - h^{-1}dt)^2 + \sum_{i=1}^3 dx_i^2 + dr^2 + r^2 ds_{B(2)}^2 \\ & + r^2 \left(\frac{(d\eta R + b)dz}{R} + (a + c\eta R)d\phi + \mathcal{A} \right)^2 + \sum_{i=8}^9 dy_i^2 . \end{aligned} \tag{4.3}$$

3. Reduce to IIA along the z direction. There are several subtleties in performing this step. At this point it is convenient to set

$$z = \frac{1}{d} \tilde{z}, \tag{4.4}$$

where \tilde{z} has the periodicity $\tilde{z} \sim \tilde{z} + 2\pi dR$, and

$$\begin{aligned}
 ds^2 = & -h^{-1}dt^2 + h(d\tilde{z} + cRd\phi - h^{-1}dt)^2 + \sum_{i=1}^3 dx_i^2 + dr^2 + r^2 ds_{B(2)}^2 \\
 & + r^2 \left(\frac{(\eta R + \frac{b}{d})d\tilde{z}}{R} + (a + c\eta R)d\phi + \mathcal{A} \right)^2 + \sum_{i=8}^9 dy_i^2 .
 \end{aligned} \tag{4.5}$$

The IIA solution after the reduction then has the form

$$\begin{aligned}
 ds^2 = & \sqrt{h + \frac{(b + d\eta R)^2 r^2}{d^2 R^2}} \left(-h^{-1}dt^2 + \sum_i dx_i^2 + dr^2 + r^2 ds_{B(2)}^2 \right. \\
 & \left. + \frac{r^2 (dhR\mathcal{A} + hRd\phi + (b + d\eta R)dt)^2}{h(d^2 hR^2 + r^2(b + d\eta R)^2)} + \sum_{i=8,9} dy_i^2 \right), \\
 A = & cRd\phi + \frac{R(b + d\eta R)r^2(d\phi + d\mathcal{A}) - d^2 Rdt}{d^2 R^2 h + (b + d\eta R)^2 r^2} \\
 e^\Phi = & \tilde{g}_{\text{IIA}} \left(h + \frac{(b + d\eta R)^2 r^2}{d^2 R^2} \right)^{3/4},
 \end{aligned} \tag{4.6}$$

for which the string coupling constant and the tension change to

$$\tilde{g}_{\text{IIA}} = d^{3/2} g_{\text{IIA}}, \quad \tilde{\alpha}' = \frac{1}{d} \alpha', \tag{4.7}$$

because the choice of M-theory circle is different.

4. T-dualize along x_1 , x_2 , and x_3 . This brings the background to the form

$$\begin{aligned}
 ds^2 = & \sqrt{h + \frac{(b + d\eta R)^2 r^2}{d^2 R^2}} \left(-h^{-1}dt^2 + \frac{1}{h + \frac{(b + d\eta R)^2 r^2}{d^2 R^2}} \sum_i dx_i^2 + dr^2 + r^2 ds_{B(2)}^2 \right. \\
 & \left. + \frac{r^2 (dhR\mathcal{A} + hRd\phi + (b + d\eta R)dt)^2}{h(d^2 hR^2 + r^2(b + d\eta R)^2)} + \sum_{i=8,9} dy_i^2 \right), \\
 A = & \left(cRd\phi + \frac{R(b + d\eta R)r^2(d\phi + d\mathcal{A}) - d^2 Rdt}{d^2 R^2 h + (b + d\eta R)^2 r^2} \right) \wedge dx_1 \wedge dx_2 \wedge dx_3, \\
 e^\Phi = & \tilde{g}_{\text{IIB}} .
 \end{aligned} \tag{4.8}$$

The radii of the x_i coordinates are

$$\tilde{R}_i = \frac{\tilde{\alpha}'}{\alpha'} R_i = \frac{R_i}{d}. \tag{4.9}$$

We also find

$$\tilde{g}_{\text{IIB}} = \tilde{g}_{\text{IIA}} \tilde{V} \tilde{\alpha}'^{-3/2} = g_{\text{IIB}}, \quad \tilde{V} = \tilde{R}_1 \tilde{R}_2 \tilde{R}_3. \tag{4.10}$$

In terms of $\tilde{\alpha}'$ and $\tilde{g}_{\text{IIB}} = g_{\text{IIB}}$,

$$h = 1 + \frac{4\pi g_{\text{IIB}} N d^2 \tilde{\alpha}'^2}{r^4}, \tag{4.11}$$

indicating that the number of D3-branes has become d^2N . However, we see that near $r = 0$, the $d\phi^2$ component of the metric has the form

$$\frac{1}{d^2}r^2(\sqrt{h})d\phi^2 \tag{4.12}$$

indicating that there is a Z_d orbifold singularity. The total D3-brane charge is therefore dN .

5. Now, we take the decoupling limit sending $\tilde{\alpha}' \rightarrow 0$ keeping $\tilde{U} \equiv r/\tilde{\alpha}'$, $\tilde{g}_{\text{IIB}} = dR\tilde{\mathcal{V}}/\tilde{\alpha}'^2$ [using (4.1), (4.7) and (4.10)], and $\chi = \eta R$. This brings the SUGRA solution to the form

$$\begin{aligned} \frac{ds^2}{\alpha'} &= \tilde{K}^{1/2} \left(-\tilde{H}^{-1}dt^2 + d\tilde{U}^2 + \tilde{U}^2 ds_{B(2)}^2 + \sum_{i=8,9} dY_i^2 \right) \\ &\quad + \tilde{K}^{-1/2} \left(\sum_i dx_i^2 + \tilde{H}\tilde{U}^2 \left(\frac{d\phi}{d} + \mathcal{A} + \Delta^3 H^{-1} dt \right)^2 \right), \\ \frac{A}{\alpha'^2} &= \left\{ \frac{c}{d} \tilde{g}_{\text{IIB}} \tilde{\mathcal{V}}^{-1} d\phi + \tilde{K}^{-1} \left(-dt + \tilde{U}^2 \tilde{\Delta}^3 \left(\frac{d\phi}{d} + \mathcal{A} \right) \right) \right\} \wedge dx_1 \wedge dx_2 \wedge dx_3, \\ e^\Phi &= \tilde{g}_{\text{IIB}}, \end{aligned} \tag{4.13}$$

where

$$\tilde{\Delta}^3 \equiv \frac{(b + d\chi)\tilde{\mathcal{V}}}{\tilde{g}_{\text{IIB}}}, \quad \tilde{H} \equiv \frac{4\pi\tilde{g}_{\text{IIB}}N}{(\tilde{U}^2 + \|Y\|^2)^2}, \quad \tilde{K} \equiv \tilde{H} + \tilde{U}^2 \tilde{\Delta}^6. \tag{4.14}$$

This is the SUGRA dual of the $SL(2, Z)$ transform of PFT. It has the same form as the SUGRA dual of PFT (2.20), except that the ϕ coordinate has a deficit angle, and there is an extra constant term in the RR 4-form potential (which cannot be gauged away because the x_1, x_2, x_3 and ϕ directions are compact).

The $SL(2, Z)$ transformation also acts non-trivially on the D3-brane charge, the volume of the torus, and the puffness. Specifically

$$N \rightarrow \tilde{N} = d^2N, \quad R_i \rightarrow \tilde{R}_i = R_i/d, \quad \chi \rightarrow \tilde{\chi} = b + d\chi, \tag{4.15}$$

which is the analogue of the Morita transformation formula of NCYM [21, 31]. Note that the UV/IR relation

$$E = \frac{\tilde{U}}{\sqrt{\tilde{\lambda}}} = \frac{U}{\sqrt{\lambda}} \tag{4.16}$$

gives rise to a consistent holographic embedding. The constant part of the RR 4-form in PFT is the analogue of the “ Φ parameter” in NCYM.

We can immediately infer the range of validity of this solution

$$\frac{\tilde{\lambda}^{-1/4}}{\tilde{R}_i} \ll E \ll \frac{\tilde{\lambda}^{-1/2} \tilde{R}_i^2}{\tilde{\Delta}^3} = \frac{\tilde{g}_{\text{IIB}} \tilde{\lambda}^{-1/2}}{\tilde{\chi}} \frac{1}{\tilde{R}_i}. \tag{4.17}$$

Computing the entropy from the near extremal generalization of (4.13) yields

$$S = \frac{1}{d} (d^2N)^2 (2\pi)^3 \tilde{\mathcal{V}} T^3 = (2\pi)^3 N^2 \mathcal{V} T^3. \tag{4.18}$$

In other words, the functional form of the entropy formula (3.8) appears to extend beyond its naive range of applicability, as long as there is some dual description in terms of one of the $SL(2, Z)$ duals listed in (4.13).

If the value of χ of PFT we start with is rational, say,

$$\chi = \frac{r}{s} \tag{4.19}$$

then there exists an element of $SL(2, Z)$,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} p & -r \\ -q & s \end{pmatrix}, \tag{4.20}$$

for which $\tilde{\Delta} = 0$, and the solution becomes

$$\begin{aligned} \frac{ds^2}{\tilde{\alpha}'} &= -\tilde{H}^{-1/2} dt^2 + \tilde{H}^{-1/2} \sum_i dx_i^2 + \tilde{H}^{1/2} \left(d\tilde{U}^2 + \tilde{U}^2 ds_{B(2)}^2 + \tilde{U}^2 \left(\frac{d\phi}{s} + \mathcal{A} \right)^2 \right) \\ \frac{A}{\tilde{\alpha}'^{3/2}} &= \left(-\frac{q}{s} \tilde{g}_{\text{IIB}} \tilde{\mathcal{V}}^{-1} d\phi - \tilde{H}^{-1} dt \right) \wedge dx_1 \wedge dx_2 \wedge dx_3 \\ e^\Phi &= \tilde{g}_{\text{IIB}} \end{aligned} \tag{4.21}$$

which is essentially $AdS_5 \times S_5/Z_s$ with constant RR 4-form potential. The fact that this solution is anti de-Sitter provides further evidence that gravity is decoupled from the dynamics of PFT.

The region of validity of (4.21) is

$$\frac{\tilde{\lambda}^{-1/4}}{\tilde{R}} < E. \tag{4.22}$$

Provided χ is chosen such that

$$1 \ll \frac{\sqrt{s}\lambda^{1/4}}{g_{\text{IIB}}\chi}, \tag{4.23}$$

which is easy to arrange since we assumed $\lambda \gg 1$ and $g_{\text{IIB}} \ll 1$, the upper bound of (4.17) is smaller than the lower bound of (4.22):

$$\frac{\lambda^{-1/2} R^2}{\Delta^3} \ll \frac{\tilde{\lambda}^{-1/4}}{\tilde{R}}. \tag{4.24}$$

As the supergravity dual (4.21) does not have an upper bound on its region of applicability (4.17), one can conclude that any PFT with rational value of χ is described in terms of it. Since rational χ 's form a dense subset of the set of real values of χ , we conclude that for arbitrary values of χ , the entropy formula (3.8) is valid for all energies,

$$\frac{\lambda^{-1/4}}{R_i} < E, \tag{4.25}$$

assuming that the entropy is a continuous function of χ . The specific $SL(2, Z)$ element which gives the most effective description at a given energy E in the range (4.25) does, however, depend sensitively on the rationality of χ . To determine which $SL(2, Z)$ is most

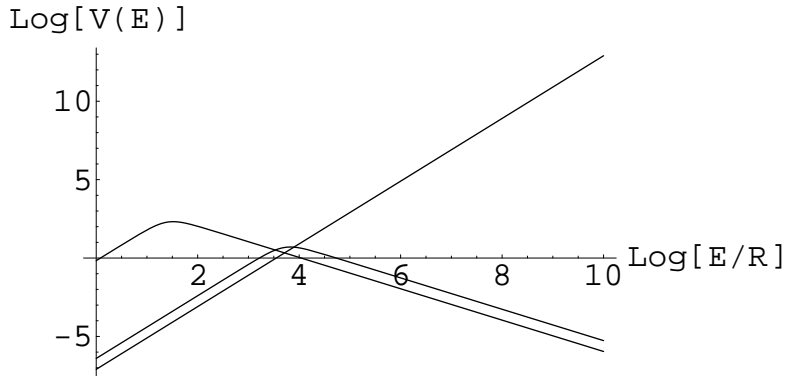


Figure 1: Log-log plot of (4.26) for $(b, d) = (0, 1)$, $(b, d) = (-1, 511)$, and $(b, d) = (-2, 1023)$ for a PFT with parameters $2\pi g_{\text{YM}}^2 = g_{\text{IIB}} = 1/3$, $\lambda = 9$, and $\chi = 2/1023$.

effective, one looks for a pair (b, d) that maximizes the proper size of the x_i circle, or equivalently the expression

$$V(E) \equiv \frac{\tilde{R}^4}{\tilde{H}^2 + \tilde{\Delta}^6 \tilde{U}^2} = \frac{g_{\text{IIB}}^2 R^2 \lambda^2 E^4}{4\pi d^2 g_{\text{IIB}}^2 + (b + d\chi)^2 \lambda^2 R^6 E^6}. \quad (4.26)$$

For example, take $g_{\text{IIB}} = 1/3$, $\lambda = 9$, and $\chi = 2/1023$. We find that $(b, d) = (0, 1)$, $(b, d) = (-1, 511)$, and $(b, d) = (-2, 1023)$ give rise to a $V(E)$ that is illustrated in figure 1. Similar structures were encountered in the case of non-commutative gauge theory [21] and NCOS [30], where a self-similar structure, closely related to the continued fraction expansion for the appropriate counterparts of the dimensionless non-locality parameter χ , characterizes the phase diagram.

Here, the fact that the range of validity (4.17) depends on two large dimensionless parameters $\tilde{\lambda}$ and $1/\tilde{g}_{\text{IIB}}$, in addition to χ , makes the full phase structure somewhat more cumbersome to determine. For example, for $\tilde{\Delta} \ll \tilde{g}_{\text{IIB}}^{1/3} \tilde{\lambda}^{-1/6} R$ [cf. (2.40), which is stronger than (2.26) for $\lambda^{1/4} \gg g_{\text{IIB}}$], we need to go to the M-theory description as we explained at the end of section 2. This description does not drastically alter the form of the entropy as a function of temperature, because the entropy formula (3.8), which is based on the area of the horizon in Einstein frame, is generally unaffected by T-dualities and by the M-theory lift. Strictly speaking, we have not ruled out the possibility of some exotic thermodynamic behavior in the range of energies for which the supergravity description is not effective, along the lines of what was observed in [30]. Nonetheless, one expects some specific dual description to be effective for any range of parameters and energies. In these duality cascades, the fact that there is a PFT in the far IR and (4.21) in the far UV for any rational value of χ appears to be a robust feature.

This also highlights the point that the decompactification limit $R_i \rightarrow \infty$ is a tricky limit to take even if one concentrates on the UV. This is because making R_i large while keeping Δ fixed changes the rationality of χ in a chaotic way. While physical observables, such as the entropy, have a smooth limit, the phase structure in the UV region evolves erratically. Such interference between flows to the UV and decompactification is a typical feature of non-local field theories [21, 30].

5. Deformation operator of lowest dimension

In the limit $U \rightarrow 0$ the supergravity dual (2.20) becomes $AdS_5 \times S^5$, which corresponds to PFT flowing to $\mathcal{N} = 4$ SYM in the IR. The supergravity dual (2.20) can also be used to read off the lowest dimension operator responsible for deforming the $\mathcal{N} = 4$ theory. We see in (2.20) that a linear combination of the metric and the RR 4-form potential, polarized partly along the brane and partly transverse to the brane, are deformed. The deformation of $AdS_5 \times S^5$ that (2.20) describes has been arranged to preserve half of the supersymmetries, and therefore the corresponding operator has to be a descendant of a chiral primary operator.

We denote the $\mathcal{N} = 4$ SYM gauge field strength by $F_{\mu\nu}$, the scalars by X^I ($I = 4, \dots, 9$ for convenience), and the spinors by λ and $\bar{\lambda}$. (We will not need to specify the indices on the spinors.) The descendants of chiral primary operators of $\mathcal{N} = 4$ SYM are listed in table 7 of [32]. In their notation, our requisite descendant takes the form

$$\mathcal{O}_k^{(17)} \sim \text{Tr} F_+ F_- \lambda \bar{\lambda} X^k \tag{5.1}$$

for $k = 0$. The schematic notation here is as follows: F_+ (F_-) stands for the self-dual (anti-self-dual) part of the field-strength, X^k stands for a product of k scalar fields, there is an unspecified index contraction, and terms involving derivatives and commutators have been suppressed. $\mathcal{O}_{k=0}^{(17)}$ is an operator of dimension 7 in the representation **15** of the $SO(6)$ R -symmetry group. This $SO(6)$ multiplet accounts for distinct ways in which the space \mathbb{R}^4 , which we twist, can be embedded into the \mathbb{R}^6 space transverse to the D3-brane.

Let us note, in contrast, that the leading irrelevant operators that deform $\mathcal{N} = 4$ SYM into SYM on a noncommutative \mathbb{R}^4 (NCYM) and the noncommutative open string theory (NCOS) are, respectively, the real and the imaginary parts of the dimension 6 operator

$$\mathcal{O}_{k=0}^{(16)} \sim \text{Tr} F_+ F_-^2 X^{k=0}, \tag{5.2}$$

whereas the dipole deformation and its S-dual are generated by the real and imaginary parts of the dimension 5 operator

$$\mathcal{O}_{k=0}^{(10)} \sim \text{Tr} F_+ \lambda \bar{\lambda} X^{k=0}. \tag{5.3}$$

These dimensions fit well with the fact that the parameters characterizing the dipole, the non-commutative, and the puff field theories have dimensions 1, 2, and 3, respectively.

PFT can also be defined as the decoupled field-theory that describes N D3-brane probes in the strongly-coupled type-IIIB background obtained from (2.14) by setting $N = 0$. This is the Melvin background that can be written as

$$\begin{aligned} ds^2 &= (1 + \eta^2 u^2)^{1/2} \left(-dt^2 + du^2 + u^2 ds_{B(2)}^2 + \sum_{i=8,9} dy_i^2 \right) \\ &\quad + (1 + \eta^2 u^2)^{-1/2} \left(\sum_i dx_i^2 + u^2 (d\phi + \mathcal{A} + \eta dt)^2 \right), \\ A &= \frac{1}{1 + \eta^2 u^2} (-dt + u^2 \eta (d\phi + \mathcal{A})) \wedge dx_1 \wedge dx_2 \wedge dx_3, \\ e^\Phi &= g_{\text{IIB}}. \end{aligned} \tag{5.4}$$

It is strongly coupled in the limit $\alpha' \rightarrow 0$ keeping (2.17). The operator $\mathcal{O}_{k=0}^{(17)}$ can be interpreted as follows. Expand (5.4) formally in powers of η , and keep only terms up to order $O(\eta)$. Using the notation

$$\omega \equiv u^2(d\phi + \mathcal{A}) = y_4 dy_5 - y_5 dy_4 + y_6 dy_7 - y_7 dy_6,$$

we can write (5.4) as

$$\begin{aligned} ds^2 &= -dt^2 + \sum_i dx_i^2 + \sum_{i=4}^9 dy_i^2 + 2\eta\omega dt + O(\eta^2), \\ A_4^{(\text{full})} &= (-dt + \eta\omega) \wedge dx_1 \wedge dx_2 \wedge dx_3 + \eta\omega \wedge dt \wedge dy_8 \wedge dy_9 + O(\eta^2), \\ e^\Phi &= g_{\text{IIB}}, \end{aligned} \tag{5.5}$$

where we have completed the RR 4-form so that $dA_4^{(\text{full})}$ is self-dual. The bosonic part of $\mathcal{O}_{k=0}^{(17)}$ can now be deduced from the Dirac-Born-Infeld (DBI) action and the Wess-Zumino (WZ) term,

$$\mathcal{S}_{\text{DBI+WZ}} = \frac{1}{\alpha'^2 g_{\text{IIB}}} \int_{D3} \left(\sqrt{-\det G + \alpha' F} + A_4^{(\text{full})} \right),$$

where the induced metric G is given by

$$G_{\mu\nu} = \eta_{\mu\nu} + \alpha'^2 \sum_{i=4}^9 \partial_\mu X^i \partial_\nu X^i + \eta\alpha'^2 (\delta_{\mu 0} \mathcal{J}_\nu + \delta_{\nu 0} \mathcal{J}_\mu) + O(\eta^2),$$

\mathcal{J}_μ is the R-current:

$$\mathcal{J}_\mu \equiv X^4 \partial_\mu X^5 - X^5 \partial_\mu X^4 + X^6 \partial_\mu X^7 - X^7 \partial_\mu X^6,$$

and we used the standard relation $y_i = \alpha' X^i$ ($i = 4 \dots 9$) between the transverse coordinates of the D3-brane and the scalar fields of the effective field theory on the brane.

Expanding the DBI action to order $O(\eta)$ we find, for $N = 1$,

$$\mathcal{O}_{k=0}^{(17)} = T^{0\mu} \mathcal{J}_\mu + \epsilon^{0\mu\nu\sigma} \partial_\mu X^8 \partial_\nu X^9 \mathcal{J}_\sigma + \text{fermions}, \tag{5.6}$$

where

$$T^{\mu\nu} = \sum_{i=4}^9 \partial^\mu X^i \partial^\nu X^i - \frac{1}{2} \eta^{\mu\nu} \sum_{i=4}^9 \partial_\tau X^i \partial^\tau X^i + F^\mu{}_\tau F^{\tau\nu} + \frac{1}{4} \eta^{\mu\nu} F_{\sigma\tau} F^{\sigma\tau} + \text{fermions},$$

is the stress-energy tensor. For $N > 1$, (5.6) is missing an overall trace and additional commutator terms.

6. Concluding remarks

In this paper we inferred a number of basic features of Puff Field Theory by analyzing its supergravity dual. In particular, we computed the thermodynamic entropy, studied its range of validity, and identified the leading irrelevant operator deforming the $\mathcal{N} = 4$

theory. These results lend more credence to the conjecture that PFT is decoupled for gravity. In fact, the mere existence of a (geodesically complete) near-horizon limit of the background (2.14) implies decoupling. The finite entropy (3.8) suggests that the spectrum is discrete (for appropriate boundary conditions that eliminate the zero modes of the low-energy scalar fields). Furthermore, we have seen that for rational χ the supergravity dual can be transformed into an orbifold of $AdS_5 \times S^5$ with extra RR flux (4.21), which certainly describes a decoupled theory.

It would, of course, be interesting if a microscopic definition of PFT can be found. Non-commutative Yang-Mills theory and dipole theories can be formulated in terms of a concrete action, and NCOS can be defined as a strong coupling limit of NCYM. It would be nice if PFT can be defined at the same level of specificity.

Lessons from NCYM and dipole theories suggest that a good starting point might be to study PFT on T^3 with a rational parameter χ . One approach might be to identify the field theory dual of (4.21). This rather innocent looking supergravity solution contains a closed RR 4-form potential which, combined with the orbifold, is responsible for all the non-trivial IR physics. We are currently investigating this issue and we hope to report our findings in the near future.

PFT arose as the decoupling limit of D0-branes in a Melvin universe supported by an RR 1-form potential in the type IIA theory. It is also natural to consider what happens for other type-IIA Dp -branes.

For the case of D2-branes, there are two possible choices of embedding: the twisted ϕ coordinate could either be along or transverse to the D2-brane. If it is along the brane, one ends up with an NCOS, which is S-dual to the non-commutative gauge theory of [8, 9], dimensionally reduced to 2+1 dimensions. The supergravity dual of the NCOS (prior to the dimensional reduction) was discussed in [33, 34]. If the ϕ direction is transverse to the brane, we end up with the S-dual of dipole theory. In both of these constructions, we are dimensionally reducing along the non-local direction from the NCYM/dipole point of view, but the non-locality of the S-dual survives dimensional reduction.

The case of D4-branes does not appear to have any interesting non-local field theory in the decoupling limit, because when D4-branes are lifted to M-theory they are extended along the M-theory circle. The case of NS5-branes appears to lead to a non-local deformation of little string theory (LST), and neither D6-branes nor D8-branes support any decoupled field theory, so we will not pursue them further.

We will elaborate on the details of the twisted decoupling of type IIA D2, D4, and NS5-branes in appendix A. As NCOS arising from 2+1 was already known, and LST arising from NS5 is already a non-local theory, the PFT based on D0-branes appears to be rather special in giving rise to a novel non-local deformation of a local field theory.

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A. Melvin twists of D2, D4, and NS5 branes

In this article, we primarily focused on the decoupled field theory on D0-branes embedded in a Kaluza-Klein Melvin universe with the M-theory circle playing the role of the Kaluza-Klein circle. Such a construction naturally extends to other branes in type-IIA string theory. In this appendix, we elaborate on the cases of D2, D4, and NS5 branes. In all of these cases, the appropriate scaling of the Melvin flux can be inferred from requiring the dimensionless parameter χ to be finite.

A.1 D2-brane

The decoupled theory on D2-branes turns out to be a known non-local field theory. In order to identify this field theory, let us analyze the supergravity dual explicitly.

Let us follow the construction of section 2. Start with the supergravity solution of D2

$$\begin{aligned}
 ds^2 &= h^{-1/2} \left(-dt^2 + \sum_{i=1}^2 dx_i^2 \right) + h^{1/2} \sum_{i=3}^9 dx_i^2, \\
 A &= h^{-1} dt \wedge dx_1 \wedge dx_2, \\
 e^\Phi &= g_{\text{IIA}} h^{1/4}, \\
 h &= 1 + \frac{6\pi^2 g_{\text{IIA}} N \alpha'^{5/2}}{r^5}.
 \end{aligned}
 \tag{A.1}$$

Lifting to M-theory gives

$$\begin{aligned}
 ds^2 &= h^{-2/3} \left(-dt^2 + \sum_{i=1}^2 dx_i^2 \right) + h^{1/3} \sum_{i=3}^9 dx_i^2 + h^{1/3} dz^2, \\
 A &= h^{-1} dt \wedge dx_1 \wedge dx_2, \\
 z &\sim z + 2\pi g_{\text{IIA}} \ell_s.
 \end{aligned}
 \tag{A.2}$$

Twisting along the (x_1, x_2) plane gives

$$\begin{aligned}
 ds^2 &= h^{-2/3} \left(-dt^2 + d\rho^2 + \rho^2 (d\phi + \eta dz)^2 \right) + h^{1/3} \sum_{i=3}^9 dx_i^2 + h^{1/3} dz^2, \\
 A &= h^{-1} \rho dt \wedge d\rho \wedge (d\phi + \eta dz), \\
 z &\sim z + 2\pi g_{\text{IIA}} \ell_s.
 \end{aligned}
 \tag{A.3}$$

Now, reduce to IIA on z to find

$$\begin{aligned}
 ds^2 &= \left(\frac{h + \eta^2 \rho^2}{h} \right)^{1/2} \left[h^{-1/2} \left(-dt^2 + d\rho^2 + \frac{h\rho^2}{h + \eta^2 \rho^2} d\phi^2 \right) + h^{1/2} (dr^2 + r^2 d\Omega_6^2) \right], \\
 A_1 &= \frac{\eta \rho^2}{h + \eta^2 \rho^2} d\phi, \\
 A_3 &= h^{-1} dt \wedge dx_1 \wedge dx_2, \\
 B_2 &= \eta h^{-1} r dt \wedge dr, \\
 e^\Phi &= g_{\text{IIA}} h^{1/4} \left(\frac{h + \eta^2 \rho^2}{h} \right)^{3/4}.
 \end{aligned} \tag{A.4}$$

Finally, taking the $\alpha' \rightarrow 0$ decoupling limit, keeping $U = r/\alpha'$, $g_{\text{YM2}}^2 = g_{\text{IIA}} \ell_s^{-1}$ (the YM coupling constant of the 2+1D theory) and $\chi = \eta R$ fixed, gives

$$\begin{aligned}
 \frac{ds^2}{\alpha'} &= \left(1 + \frac{\chi^2 \rho^2}{g_{\text{YM2}}^4 H} \right)^{1/2} \left[H^{-1/2} \left(-dt^2 + d\rho^2 + \frac{\rho^2 d\phi^2}{1 + \frac{\chi^2 \rho^2}{g_{\text{YM2}}^4 H}} \right) + H^{1/2} (dU^2 + U^2 d\Omega_6^2) \right], \\
 \frac{A_1}{\alpha'} &= \frac{\chi g_{\text{YM2}}^2 \rho^2}{g_{\text{YM2}}^4 H + \chi^2 \rho^2} d\phi, \\
 \frac{A_3}{\alpha'^2} &= H^{-1} dt \wedge dx_1 \wedge dx_2, \\
 \frac{B_2}{\alpha'} &= \frac{1}{g_{\text{YM2}}^2} \chi H^{-1} \rho dt \wedge d\rho, \\
 e^\Phi &= g_{\text{YM2}}^2 H^{1/4} \left(\frac{g_{\text{YM2}}^4 H + \chi^2 \rho^2}{g_{\text{YM2}}^4 H} \right)^{3/4}, \\
 H &= \frac{6\pi^2 g_{\text{YM2}}^2 N}{U^5}.
 \end{aligned} \tag{A.5}$$

This is a non-local deformation of a strongly coupled SYM with 16 supercharges in 2+1 dimensions.

In order to bring this theory into context, it is useful to compactify x_3 on a circle of radius α'/R_3 and smear, so that (A.5) becomes

$$\begin{aligned}
 \frac{ds^2}{\alpha'} &= \left(1 + \frac{\chi^2 \rho^2}{g_{\text{YM2}}^4 H} \right)^{1/2} \left[H^{-1/2} \left(-dt^2 + d\rho^2 + \frac{\rho^2 d\phi^2}{1 + \frac{\chi^2 \rho^2}{g_{\text{YM2}}^4 H}} \right) \right. \\
 &\quad \left. + \frac{H^{1/2}}{\alpha'^2} dx_3^2 + H^{1/2} (dU^2 + U^2 d\Omega_5^2) \right], \\
 \frac{A_1}{\alpha'} &= \frac{\chi \rho^2}{g_{\text{YM2}}^2 H (1 + \frac{\chi^2 \rho^2}{g_{\text{YM2}}^4 H})} d\phi, \\
 \frac{A_3}{\alpha'^2} &= H^{-1} dt \wedge dx_1 \wedge dx_2, \\
 \frac{B_2}{\alpha'} &= \frac{1}{g_{\text{YM2}}^2} \chi H^{-1} \rho dt \wedge d\rho, \\
 e^\Phi &= g_{\text{YM2}}^2 H^{1/4} \left(\frac{g_{\text{YM2}}^4 H + \chi^2 \rho^2}{g_{\text{YM2}}^4 H} \right)^{3/4}, \\
 H &\equiv \frac{8\pi^2 g_{\text{YM2}}^2 R N}{U^4}.
 \end{aligned} \tag{A.6}$$

T-dualizing along x_3 brings this to the form

$$\begin{aligned}
 \frac{ds^2}{\alpha'} &= \left(1 + \frac{(2\pi R_3 \chi)^2 \rho^2}{g_{\text{YM}3}^4 H}\right)^{1/2} \left[H^{-1/2} \left(-dt^2 + d\rho^2 + \frac{\rho^2 d\phi^2 + dx_3^2}{1 + \frac{(2\pi R_3 \chi \rho)^2}{g_{\text{YM}3}^4 H}} \right) \right. \\
 &\quad \left. + H^{1/2} (dU^2 + U^2 d\Omega_5^2) \right], \\
 \frac{A_2}{\alpha'} &= \frac{2\pi R_3 \chi \rho^2}{g_{\text{YM}3}^2 H \left(1 + \frac{(2\pi R_3 \chi)^2 \rho^2}{g_{\text{YM}3}^4 H}\right)} d\phi \wedge dx_3, \\
 \frac{A_4}{\alpha'^2} &= H^{-1} dt \wedge dx_1 \wedge dx_2 \wedge dx_3, \\
 \frac{\tilde{B}_2}{\alpha'} &= \frac{1}{g_{\text{YM}3}^2} (2\pi R_3 \chi) H^{-1} \rho dt \wedge d\rho, \\
 e^\Phi &= g_{\text{YM}3}^2 \left(1 + \frac{(2\pi R_3 \chi)^2 \rho^2}{g_{\text{YM}3}^4 H}\right)^{1/2}, \\
 H &= \frac{4\pi g_{\text{YM}3}^2 N}{U^4}, \\
 g_{\text{YM}3}^2 &= 2\pi R_3 g_{\text{YM}2}^2.
 \end{aligned} \tag{A.7}$$

(Here A_4 is not the complete RR 4-form, but is such that the 5-form RR field-strength is the self-dual part of dA_4 .) This solution is the S-dual of the solution of [8, 9], which was also discussed in [33, 34]. In other words, (A.5) can be viewed as a dimensional reduction of NCOS from 3+1 to 2+1 dimensions.

Twisting instead along a direction transverse to the D2-brane gives rise to the S-dual of dipole theories [11, 12], dimensionally reduced from 3+1 to 2+1 dimensions along similar lines.

A.2 D4-brane

A similar construction applied to the case of a D4-brane turns out *not* to give rise to any interesting non-local field theory. One reason for this is the fact that a D4-brane, when lifted to M-theory, wraps the M-theory circle unlike the D0 and the D2-branes. Also, the radius of the M-theory circle remains finite in the decoupling limit, as can be seen from the relation

$$R = g_{\text{IIA}} \alpha'^{1/2} = g_{\text{YM}4}^2 = \text{finite}. \tag{A.8}$$

The decoupled theory turns out to be nothing more than a twisted compactification of the decoupled M5 superconformal field theory.

To see this more explicitly, start with the supergravity solution of the D4-brane¹

$$\begin{aligned}
 ds^2 &= h^{-1/2} (-dt^2 + \sum_{i=1}^4 dx_i^2) + h^{1/2} \sum_{i=5}^9 dx_i^2, \\
 e^\Phi &= g_{\text{IIA}} h^{-1/4}, \\
 h &= 1 + \frac{gN\alpha'^{3/2}}{r^3}.
 \end{aligned} \tag{A.9}$$

¹We abbreviate the RR 2-form potential as it plays no significant role here.

This solution lifts to M-theory as follows

$$ds^2 = h^{-1/3} \left(-dt^2 + \sum_{i=1}^4 dx_i^2 + dz^2 \right) + h^{2/3} \sum_{i=5}^9 dx_i^2 . \quad (\text{A.10})$$

Now, twist along the world volume,

$$ds^2 = h^{-1/3} \left(-dt^2 + d\rho^2 + \rho^2 \left\{ ds_{B(2)}^2 + (d\phi + \mathcal{A} + \eta dz)^2 \right\} + dz^2 \right) + h^{2/3} \sum_{i=5}^9 dx_i^2 , \quad (\text{A.11})$$

and reduce to IIA,

$$ds^2 = \frac{1}{\sqrt{1+\eta^2 r^2}} \left(h^{-1/2} \left[-dt^2 + d\rho^2 + \rho^2 \left\{ ds_{B(2)}^2 + \frac{1}{1+\eta^2 \rho^2} (d\phi + \mathcal{A})^2 \right\} \right] + h^{1/2} \sum_{i=5}^9 dx_i^2 \right) . \quad (\text{A.12})$$

In the decoupling limit where $R = g_{\text{IIA}} \ell_s = g_{YM4}^2$ is kept fixed, η also stays finite. This leads to a relatively boring field theory which is nothing more than 4+1 SYM in a Kaluza-Klein Melvin universe, which lifts to a M5 SCFT on flat 5+1 dimensional space-time with twisted compactification

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 \left\{ ds_{B(2)}^2 + (d\phi + \mathcal{A} + \eta dz)^2 \right\} + dz^2 , \quad z \sim z + 2\pi R . \quad (\text{A.13})$$

A.3 NS5-brane

The Kaluza-Klein Melvin background also gives rise to a UV deformation for the decoupled theory on NS5-branes. To see this, start with the supergravity solution for NS5-branes²

$$\begin{aligned} ds^2 &= -dt^2 + \|d\vec{x}\|^2 + h(r) \left(dr^2 + r^2 \left\{ ds_{B(2)}^2 + (d\phi + \mathcal{A})^2 \right\} \right) , \\ e^\Phi &= g_s h(r)^{1/2} , \\ h(r) &= 1 + \frac{m\alpha'}{r^2} . \end{aligned} \quad (\text{A.14})$$

Lifting to M-theory, we find

$$ds^2 = h^{-1/3} (-dt^2 + \|d\vec{x}\|^2) + h^{2/3}(r) \left(dr^2 + r^2 \left\{ ds_{B(2)}^2 + (d\phi + \mathcal{A})^2 + dz^2 \right\} \right) , \quad (\text{A.15})$$

with $z \sim z + 2\pi R$. Twisting gives

$$ds^2 = -h^{-1/3} (-dt^2 + \|d\vec{x}\|^2) + h^{2/3}(r) \left(dr^2 + r^2 \left\{ ds_{B(2)}^2 + (d\phi + \mathcal{A} + \eta dz)^2 + dz^2 \right\} \right) . \quad (\text{A.16})$$

Reducing to IIA then gives

$$ds^2 = \sqrt{1 + \eta^2 r^2} \left[-dt^2 + \|d\vec{x}\|^2 + h(r) \left(dr^2 + r^2 \left\{ ds_{B(2)}^2 + \frac{(d\phi + \mathcal{A})^2}{1 + \eta^2 r^2} \right\} \right) \right] . \quad (\text{A.17})$$

In terms of $\chi = \eta g_{\text{IIA}} \ell_s$, which we are keeping finite,

$$ds^2 = \sqrt{1 + \frac{\chi^2 r^2}{g_{\text{IIA}}^2 \alpha'}} \left[-dt^2 + \|d\vec{x}\|^2 + \left(1 + \frac{N\alpha'}{r^2} \right) \left(dr^2 + r^2 \left\{ ds_{B(2)}^2 + \frac{(d\phi + \mathcal{A})^2}{1 + \chi^2 r^2 / g_{\text{IIA}}^2 \alpha'} \right\} \right) \right] . \quad (\text{A.18})$$

²We ignore the NSNS 2-form potential here as well.

If we now let $r = g_{\text{IIA}}\rho$, and send $g_{\text{IIA}} \rightarrow 0$, keeping α' fixed, to derive the dual of the decoupling limit

$$ds^2 = \sqrt{1 + \chi^2 \rho^2 \alpha'} \left[-dt^2 + \|d\vec{x}\|^2 + \frac{N\alpha'}{\rho^2} \left(d\rho^2 + \rho^2 \left\{ ds_{B(2)}^2 + \frac{(d\phi + \mathcal{A})^2}{1 + \chi^2 \rho^2 / \alpha'} \right\} \right) \right]. \tag{A.19}$$

This is a UV deformation of little string theory in type IIA with string tension α' . This solution reduces to the Callan-Harvey-Strominger solution [35] in the limit $\chi \rightarrow 0$. However, if the deformation parameter χ is rational, there exists an $\text{SL}(2, Z)$ transformation along the lines of what is described in section 4 that brings the supergravity solution to the form of a discrete orbifold of CHS with a constant RR 1-form potential.

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